Incompressive Cake Filtration: Mechanism, Parameters, and Modeling

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Improved models developed verify interpretation and prediction of incompressible filter cake thickness, filtrate volume, and rate data for linear and radial filtration cases at static and dynamic conditions. Based on the methods for determining model parameters from experimental filtration data, three sets of experimental data were analyzed, which demonstrate the diagnostic and predictive capabilities of the model. These models provide insight into the mechanism of incompressible cake filtration and offer practical means of interpreting experimental data, estimating the model parameters and simulating the linear and radial filtration processes.

Introduction

Many industrial filtration processes facilitate plate and drum filters for slurry filtration. Instruments used for various purposes may utilize porous plate, tube, and hollow disk type filters. Linear and radial filtration models are needed for prediction, simulation, and optimization of the various filtration processes, and analysis and interpretation of experimental data. Simple models, such as those presented in this study, are preferred in many applications, because of their convenience and the reduced computational effort.

The applicability of the majority of the previously reported simple analytical models, such as by Collins (1961), Hermia (1982), and de Nevers (1992), are usually limited to linear and constant rate filtration. However, models for constant pressure filtration are also required for certain applications. When the filter cake and the filter thicknesses are small relative to the radius of the filter surface exposed to slurry, linear filtration models can sufficiently approximate radial filtration. Otherwise, radial filtration processes can be better described in radial geometry.

In the following, improved simple filtration models for radial and linear incompressible cake filtration processes, with additional features such as considering variable rate, effect of filter thickness, and static and dynamic filtration conditions, are presented and verified by means of the simplified analytical models and the reported experimental data.

Formulation

For the derivation of the radial filtration model, consider that the slurry is applied to the inner surface and the filtrate leaves from the outer surface of a hollow cylindrical filter. However, the model is also valid for the reverse operation. Consider incompressible particles, carrier fluid, and filter cake, no small particle invasion and deposition within the filter cake, and a particle free filtrate. The inner and outer radii (cm) of the hollow cylindrical filter are denoted, respectively, by r_w and r_e. The radius of the slurry side cake surface is denoted by r_c. The thickness and width (cm) of the cake are denoted by δ and h, respectively. The cross-flow (or tangential) velocity of the slurry flowing over the inner filter surface (cm/s) is denoted by ν and the filtration velocity of the particle free carrier fluid normal to the inner filter surface (cm/s) by u. The filtration conditions are referred to as the static when $\nu = 0$ and the dynamic when $\nu > 0$.

The volumetric balance of the cake forming particles is given by (Civan, 1994, 1996)

$$-\epsilon_s dr_c/dt = R_{ps}/\rho_p \quad (1)$$

in which the volume fraction of the cake forming particles ϵ_s in terms of the cake porosity ϕ_c is given by

$$\epsilon_s = 1 - \phi_c \quad (2)$$

t is time(s), ρ_p is the particle density (g/cm^3), and R_{ps} denotes the net rate of particle deposition over the progressing cake surface (gm/s/cm^2).

The present problem deals with a forced filtration during the flow of slurry over the filter surface at a sufficiently rapid rate, such as in the case of the filtration of the drilling mud slurry in wells. In this case, deposition of particles is directly related to the filtration mass flux of particles of the slurry approaching the filter, colloidal detachment is negligible because there is no alteration of the physico-chemical conditions in ordinary filtration, and erosion of particles from the cake surface by the hydrodynamic shear force is dominant by several orders of magnitude. Particles can only be detached if the prevailing fluid shear exceeds the critical, minimum shear stress necessary to dislocate the particles from the cake surface. It takes a certain minimum shear stress to dislocate particles from their places on the cake surface, especially if the particles are sticky. Peng and Peden (1992) omitted this. It is reasonable to assume that the erosion rate is proportional with the excess of the shear stress above the minimum critical shear stress necessary for detachment of the cake particles from the filter cake surface. Therefore, the net mass rate of deposition of particles per unit area of the cake surface is given by the difference between the deposition and the erosion rates as (Civan, 1994, 1996)

$$R_{ps} = k_d u_c c_p - k_e(\tau - \tau_{cr}) \tag{3}$$

where u_c is the carrier fluid filtration velocity normal to the slurry side cake surface (cm/s). c_p is the slurry particle concentration expressed as the mass of particles per unit volume of the carrier fluid in the slurry (g/cm^3 carrier fluid). For dynamic filter cake formation, the non-Newtonian slurry fluid wall shear-stress (dyne/cm^2) τ is given by the Rabinowitsch-Mooney equation (Metzner and Reed, 1955)

$$\tau = k'(4v/r_c)^{n'} \tag{4}$$

where k' and n' are the consistency constant (dyne/cm^2/s$^{n'}$) and flow index (dimensionless), respectively. For Newtonian slurry fluids, $k' = \mu$ where μ is the slurry viscosity (cp) and $n' = 1$. k_d and k_e are the particle deposition (dimensionless) and erosion rate constants (s/cm), and τ_{cr} is the minimum critical fluid shear stress necessary to dislocate the particles from the cake surface (dyne/cm^2). Therefore, set $k_e = 0$ for $\tau < \tau_{cr}$. For static filter cake formation $\nu = 0$, and, therefore, $\tau = 0$. Hence, the second term on the right of Eq. 3 drops out.

The deposition and erosion rate constants depend on the properties of the particles and carrier fluid, and the conditions of the slurry, such as particle concentration, flow rate, and pressure.

Ravi et al. (1992) have determined that the following equation proposed by Potanin and Uriev (1991) predicts the critical shear stress with the same order of magnitude accuracy of their experimental measurements

$$\tau_{cr} = H/(24dl^2) \tag{5}$$

where $H = 3.0 \times 10^{-13}$ erg is the Hamaker coefficient (erg), d is the average particle diameter, and l is the separation distance (cm) between the particle surfaces in the filter cake. However, the values calculated from Eq. 5 can only be used as a first-order accurate estimate because Eq. 5 has been derived from an ideal theory. The actual value can be different, because the ideal theory does not take into account the effect of the other factors, such as aging (Ravi et al., 1992), surface roughness, and particle stickiness (Civan, 1996) on the particle detachment. Therefore, the actual value of the critical shear stress may be several fold different than that predicted by Eq. 5 using the particle size and separation distance data. Hence, Ravi et al. (1992) recommend experimental determination of the critical shear stress.

The filter cake thickness δ (cm) is the difference between the radius, r_w of the filter surface exposed to the slurry, and the filter cake radius r_c

$$\delta = r_w - r_c \tag{6}$$

Thus, substituting Eqs. 2 and 3 into Eq. 1 yields (Civan, 1994, 1996)

$$-\frac{dr_c}{dt} = \frac{k_d u_c c_p - k_e(\tau - \tau_{cr})}{(1-\phi_c)\rho_p}, t > 0 \tag{7}$$

in which set $k_e = 0$ when $\tau < \tau_{cr}$.

The cake formation begins at the filter surface exposed to the slurry, and, therefore, the initial cake radius is given by

$$r_c = r_w, \quad t = 0 \tag{8}$$

The volumetric flux of the particle free carrier fluid, that is, the filtrate, into the cake surface is given by

$$u_c = \frac{q}{2\pi r_c h} \tag{9}$$

where q is the particle free carrier fluid filtrate flow rate (cm^3/s).

Applying Eq. 9, Eq. 7 can be written as

$$-\frac{d(r_c/r_w)}{dt} = A\frac{q}{(r_c/r_w)} - B, 0 \le r_c/r_w \le 1 \tag{10}$$

where

$$A = \frac{k_d c_p}{(1-\phi_c)\rho_p 2\pi h r_w^2} \tag{11}$$

and

$$B = \frac{k_e(\tau - \tau_{cr})}{(1-\phi_c)\rho_p r_w} \text{ for } \tau > \tau_{cr} \text{ and } B = 0 \text{ for } \tau \le \tau_{cr} \tag{12}$$

The volumetric flux of the particle free filtrate is given by Darcy's law

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial r} = \frac{q}{2\pi r h} \tag{13}$$

Note that the volumetric rates of fluids flowing through the filter cake and filter media in series are the same, irrespective of whether it is a constant rate or a constant pressure filtration. In constant pressure filtration, the rate varies but the rates of fluid flowing through the filter cake and the filter media are equal at all times. In constant rate filtration, the slurry pressure varies when the filter effluent side pressure is kept constant. Thus, even when q varies temporally, its value remains spatially the same over the radial distance at any given time. Therefore, Eq. 13 can be integrated over the radial distance $r_c \leq r \leq r_e$ before and during the cake buildup from the slurry to the effluent sides to obtain the following pressure over balance expressions, respectively

$$p_c - p_e = \frac{q_o \mu}{2\pi K_f h} \ln\left(\frac{r_e}{r_w}\right) \tag{14}$$

$$p_c - p_e = \frac{q\mu}{2\pi h}\left[\frac{1}{K_c}\ln\left(\frac{r_w}{r_c}\right) + \frac{1}{K_f}\ln\left(\frac{r_e}{r_w}\right)\right] \tag{15}$$

where p_c and p_e are the pressures of the slurry and the filter outlet (atm), respectively. q_o and q are the flow rates before and during the filter cake buildup (cm^3/s). K_c and K_f are the filter cake and filter permeabilities (darcy).

Thus, equating Eqs. 14 and 15 and rearranging leads to

$$\ln(r_w/r_c) = (q_o/q - 1)(K_c/K_f)\ln(r_e/r_w) \tag{16}$$

Equation 16 can be written as

$$r_c/r_w = \exp(-C/q + D) \tag{17}$$

where

$$C = q_o D/K_f \tag{18}$$

for which q_o is determined by Eq. 14 and

$$D = (K_c/K_f)\ln(r_e/r_w) \tag{19}$$

Thus, substituting Eq. 16 into Eq. 13 and rearranging yields the filtration flow rate equation as

$$dq/dt = (-1/C)q^2[Aq\exp(C/q - D) - B]\exp(C/q - D) \tag{20}$$

subject to the initial condition given by

$$q = q_o, \quad t = 0 \tag{21}$$

where q_o is the injection rate given by Eq. 14 before the filter cake buildup.

The wall shear-stress is calculated by Eq. 4 for the varying cake radius $r_c = r_c(t)$. The filter cake thickness is calculated by means of Eqs. 6 and 17. Equations 20 and 21 can be solved numerically using an appropriate method such as the Runge-Kutta method. However, for thin cakes, it is reasonable to assume that the wall-shear stress is approximately constant because $r_c \cong r_w$. Then, Eq. 20 can be integrated as

$$t = -C\int_{q_o}^{q}\{q^2\exp(C/q - D)[Aq\exp(C/q - D) - B]\}^{-1}dq \tag{22}$$

which can be numerically evaluated by the Gaussian quadrature rule.

The radial flow equations derived above can be readily converted to a linear one-dimensional case of laboratory core tests by means of the following transformation

$$x = r^2 \tag{23}$$

Thus, the equations corresponding to Eqs. 14, 15, 17–19, 9–12 and 20–22 are obtained, respectively, as

$$p_c - p_e = q_o \mu L_f/(aK_f) \tag{24}$$

$$p_c - p_e = \left(1 + \frac{K_f \delta}{K_c L_f}\right)\frac{q\mu L_f}{aK_f} \tag{25}$$

where a is the cross-sectional area of the filter (cm^2).

Equating Eqs. 24 and 25 and rearranging, the filter cake thickness is given by

$$\delta = C/q - D \tag{26}$$

where

$$C = q_o D \tag{27}$$

$$D = L_f K_c/K_f \tag{28}$$

$$u_c = q/a \tag{29}$$

The filter cake volumetric balance leads to

$$d\delta/dt = Aq - B \tag{30}$$

where

$$A = k_d c_p/[(1-\phi_c)\rho_p a] \tag{31}$$

$$B = k_e(\tau - \tau_{cr})/[(1-\phi_c)\rho_p] \tag{32}$$

in which the shear-stress for linear filtration is given by (Metzner and Reed, 1955)

$$\tau = k'(8\nu)^{n'} \tag{33}$$

Invoking Eq. 26 into Eq. 30 leads to

$$dq/dt = -(1/C)q^2(Aq - B) \tag{34}$$

subject to

$$q = q_o, \quad t = 0 \tag{35}$$

The analytical solution of Eqs. 34 and 35 is

$$t = -\frac{C}{B}\left\{\frac{A}{B}\ln\left[\frac{\frac{A}{B} - \frac{1}{q}}{\frac{A}{B} - \frac{1}{q_o}}\right] + \frac{1}{q} - \frac{1}{q_o}\right\} \tag{36}$$

The cumulative filtrate volume is calculated by

$$Q = \int_0^t q dt \tag{37}$$

Eliminating q between Eqs. 26 and 36 yields another expression as

$$t = -\frac{C}{B}\left\{\frac{A}{B}\ln\left[\frac{\frac{A}{B}-\frac{\delta+D}{C}}{\frac{A}{B}-\frac{\delta_o+D}{C}}\right]+\frac{\delta-\delta_o}{C}\right\} \tag{38}$$

in which, usually, $\delta_o = 0$ at $t = 0$, that is, no initial filter cake.

Equation 38 is different from Eq. 7-94 of Collins (1961), because Collins did not consider the filter cake erosion. Therefore, Collins' equation applies for static filtration. To obtain Collins' result, $k_e = 0$ or $B = 0$ must be substituted in Eq. 30. Thus, eliminating q between Eqs. 26 and 30, and then integrating, yields the following equation for the filter cake thickness

$$(1/2)\delta^2 + D\delta = ACt \tag{39}$$

which results in Eq. 7-94 of Collins (1961) by invoking Eqs. 24, 27, 28, and 31 and expressing the mass of suspended particles per unit volume of the carrier fluid in terms of the volume fraction σ_p of the particles in the slurry according to

$$c_p = \rho_p \sigma_p/(1-\sigma_p) \tag{40}$$

Collins (1961) did not derive the expressions for the filtrate flow rate and the cumulative filtrate volume. These expressions can be readily obtained by integrating Eq. 34 for $B = 0$ and applying Eq. 37, respectively, as

$$q = q_o/\sqrt{1+(2Aq_o^2/C)t} \tag{41}$$

and

$$Q = (C/A)(q^{-1} - q_o^{-1}) \tag{42}$$

Equation 41 expresses that the filtrate rate declines by time due to static filter cake buildup. Donaldson and Chernoglazov (1987) represented such rate declines by an empirical decay function

$$q = q_o \exp(-\beta t) \tag{43}$$

in which β is an empirically determined coefficient.

The filtration equations presented by Hermia (1982) and de Nevers (1992) can be derived by simplifying the improved model developed in this work. For constant rate (q) filtration, Eq. 37 yields

$$Q = qt \tag{44}$$

Thus, invoking Eq. 44 into Eq. 42 results in the conventional filtration equation as

$$\frac{t}{Q} = \frac{A}{C}Q + \frac{1}{q_o} \tag{45}$$

Equation 45 is similar to Eq. 12 of de Nevers (1992) and Eq. 21 of Hermia (1982).

As can be seen by these exercises, the present improved model can be simplified to reproduce the models given by Collins (1961), Hermia (1982), and de Nevers (1992) when the inherent simplifying assumptions of these models are introduced into the present model.

Determination of Model Parameters and Diagnostic Charts

The majority of the reported filtration studies have not made attempts at measuring a full set of measurable parameters. The filtration models presented in this study may provide some guidance for the types of parameters which are needed for simulation studies.

As listed in Table 1, the present filtration models require the values of 20 parameters for simulation. Only five of these parameters may not be directly or conveniently measurable with the conventional techniques. These are the permeability K_c and porosity ϕ_c of the filter cake, the deposition and erosion rate constants k_d and k_e, and the critical shear-stress τ_{cr} for the particles. However, given the experimental measurements of the filtrate volume Q (cm^3) or rate q and the filter cake thickness δ as functions of the filtration time t, some of these parameters can be determined by means of the diagnostic charts constructed as described in the following. These are presented separately for the linear and radial filtration processes.

Linear filtration

A plot of Eq. 30 for $d\delta/dt$ vs. q yields a straight line. Substituting the slope (A) and intercept ($-B$) of this line into Eqs. 31 and 32 yields, respectively, the following expressions for the particle deposition and erosion rate constants

$$k_d = Aa(1-\phi_c)\rho_p/c_p \tag{46}$$

$$k_e = B(1-\phi_c)\rho_p/(\tau - \tau_{cr}) \tag{47}$$

In dynamic filtration, the filter cake thickness attains a certain limit value δ_∞ when the particle deposition and erosion rates equate. Simultaneously, the filtration rate also reaches a limiting value, determined by Eq. 30 as

$$q_\infty = B/A \tag{48}$$

At this condition, Eq. 26 yields the limiting value of the filter cake thickness as

$$\delta_\infty = C/q_\infty - D \tag{49}$$

Consequently, substituting Eqs. 27, 28, 31, and 32 for A, B, C, and D into Eqs. 48 and 49 leads to the following relationships for the cake permeability and the ratio of the erosion and deposition rate constants, respectively, as

Table 1. Data for Laboratory Filtration Applications

Parameters	Radial Flow Fisk et al. (1991)	Linear Flow Jiao and Sharma (1994)	Linear Flow Willis et al. (1983)
Suspension type	Seawater-based partially hydrolized polyacrylamide drilling mud	Fresh water bentonite suspension	Lucite in water suspension
Filter permeability, K_f (darcy)	6**	0.104†	—
Cake permeability, K_c (darcy)	1.35×10^{-6}	2.1×10^{-4}	—
Cake porosity, ϕ_c	0.40*	0.40*	0.388‡
Filter length, L_f (cm)	—	20.34†	—
Filter diameter, D (cm)	—	2.54†	—
Slurry injection side filter radius, r_w (cm)	2.5**	—	—
Filtrate outlet side filter radius, r_e (cm)	3.8**	—	—
Filter width, h (cm)	1.9**	—	—
Filtrate density, ρ_w (g/cm^3)	1.0	1.0	0.997‡
Particle density, ρ_p (g/cm^3)	2.5*	2.5*	1.18‡
Particle mass per carrier fluid volume, c_p (g/cm^3)	0.56**	0.04†	0.055‡
Deposition rate constant, k_d	1.1	4.3	—
Erosion rate constant, k_e (s/cm)	3×10^{-6}	7.4×10^{-7}	—
Critical shear-stress, τ_{cr} (dyne/cm^2)	0.5	5.0	—
Filtrate (water) viscosity, μ (cp)	1.0	1.0	0.969‡
Consistency constant, k' (dyne/cm^2/s$^{n'}$)	8.0†	8.0†	—
Flow index, n'	0.319†	0.319†	—
Slurry tangential velocity, ν (cm/s)	125.**	8.61†	—
Slurry application pressure, p_c (atm)	34.**	6.89†	1.7‡
Filter outlet side back pressure, p_e (atm)	1	1	1 ‡

*Data assumed.
**Data from Fisk et al. (1991).
†Data from Jiao and Sharma (1994).
‡Data from Willis et al. (1983).

$$K_c = \delta_\infty K_f / [L_f(q_o/q_\infty - 1)] \tag{50}$$

$$k_e/k_d = c_p q_\infty / [a(\tau - \tau_{cr})] \tag{51}$$

Equation 34 can be rearranged in a linear form as

$$-\frac{d}{dt}\left(\frac{1}{q}\right) = \frac{1}{q^2}\frac{dq}{dt} = -\frac{A}{C}q + \frac{B}{C} \tag{52}$$

Thus, the intercept (B/C) and slope $(-A/C)$ of the straight-line plot of Eq. 52 can be used with Eqs. 27, 28, 31, and 32 to obtain the following expressions

$$\frac{k_e}{k_d} = \frac{(B/C)c_p}{(A/C)(\tau - \tau_{cr})a} \tag{53}$$

$$k_d = \frac{(A/C)(1-\phi_c)\rho_p a q_o q_\infty \delta_\infty}{c_p(q_o - q_\infty)} \tag{54}$$

Comparing Eqs. 51 and 53 yields an alternative expression for determination of the limit filtrate rate as

$$q_\infty = (B/C)/(A/C) \tag{55}$$

Equation 55 can be used to check the value of q_∞ obtained by Eq. 48. Equation 50 can be used to determine the filter cake permeability K_c. Equations 46 and 51 or 53 and 54 can be used to calculate the particle deposition and erosion rates k_d and k_e, if the cake porosity ϕ_c and the critical shear stress τ_{cr} are known. ϕ_c can be measured. τ_{cr} can be estimated by Eq. 5, but the ideal theory may not yield a correct value as explained previously by Ravi et al. (1992) and in this article. Therefore, Ravi et al. (1992) suggested that τ_{cr} should be measured directly.

Radial filtration

Given the filter cake thickness δ, the progressing surface cake radius r_c can be calculated by Eq. 6. Then a straight line plot of $\ln(r_c/r_w)$ vs. $(1/q)$ data according to Eq. 17 yields the values of C and D as the slope and intercept of this line, respectively. A straightline plot of $[-d/dt(r_c/r_w)]$ vs. $[q/(r_c/r_w)]$ data according to Eq. 10 yields the values of A and B as the slope and intercept of this line, respectively. At static filtration conditions, $\nu = 0$ and $\tau = 0$ according to Eq. 4. Therefore, $B = 0$ according to Eq. 12. Consequently, substituting $B = 0$ and Eq. 18, Eq. 20 can be expressed in the following linear form

$$\ln\{d/dt[1/(2q^2)]\} = \ln[-q^{-3}\,dq/dt] = [\ln(A/C) - 2CK_f/q_o] + 2C/q \tag{56}$$

Thus, a straightline plot of $\ln[-q^{-3}\,dq/dt]$ vs. $(1/q)$ yields the values of $(2C)$ and $[\ln(A/C) - 2CK_f/q_o]$ as the slope and intercept of this line, respectively. This allows for determination of the A and C coefficients only. The determination of a full set of A, B, C, and D from Eqs. 10 and 20 requires both the filtrate flow rate (or volume) and the cake thickness vs. the filtration time data. Once these coefficients are determined, then their values can be used in Eqs. 11, 12, 18, and 19 to determine the values of the deposition and erosion rate constants k_d and k_e. The discussion of the linear filtration about the determination of τ_{cr} by Eq. 5 is valid also in the radial filtration case.

At dynamic equilibrium, the filter cake thickness and the filtrate flow rate attain certain limiting values δ_∞ and q_∞. Then, substituting Eq. 6 into Eqs. 10 and 17 yields the following relationships, respectively

$$Aq_\infty = B(1 - \delta_\infty/r_w) \tag{57}$$

$$1 - \delta_\infty/r_w = \exp(-C/q_\infty + D) \tag{58}$$

The filter cake permeability is determined by Eq. 19 as

$$K_c = DK_f/\ln(r_c/r_w) \tag{59}$$

The equations and the linear plotting schemes developed in this section allow for determination of the parameters of the filtration models, mentioned at the beginning of this section, from experimental filtrate flow rate (or volume) and/or filter cake thickness data. The remaining parameters should be either directly measured or estimated. In the following applications, the best estimates of the missing data have been determined by adjusting their values to fit the experimental data. This is an exercise similar to several other studies, including the ones by Liu and Civan (1996) and Tien et al. (1997). They have resorted to a model-assisted estimation of the parameters, because there is no direct method of measurement for some of these parameters.

Applications

The numerical solutions of the present models require the information on the characteristics of the slurries, particulates, carrier fluids, filters and filter cakes, the actual conditions of the tests conducted, and the measurements of all the system parameters and variables. The reported studies of the slurry filtration have measured only a few parameters and the filtrate volumes or rates. Therefore, to the author's knowledge, there is no one complete set of suitable data available in the literature that can be used for full-scale experimental verification of the present formulations. Here, the Willis et al. (1983) and Jiao and Sharma (1994) data for linear filtration and the Fisk et al. (1991) data for radial filtration are used, because these data provide more information than the other reported studies. The data are presented in Table 1 in consistent Darcy units, which are more convenient for flow through porous media.

Jiao and Sharma (1994) carried out linear filtration experiments using concentrated bentonite suspensions. They only measured the filtrate volume and predicted the filter cake thickness using a simple algebraic model. These data are given in their Figures 3 and 10, respectively. In Figures 1 to 3, their data are plotted according to the linear plotting schemes presented in the previous section for determination of parameters. As can be seen from these figures, the coefficients of Eqs. 52, 26, and 30 obtained by the least-squares regression method and the corresponding coefficients of regression are given, respectively, by

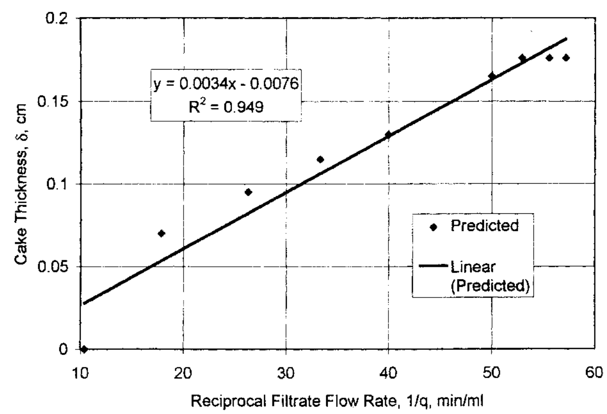

Figure 2. Correlation of Jiao and Sharma (1994) predicted filter cake thickness data according to Eq. 26.

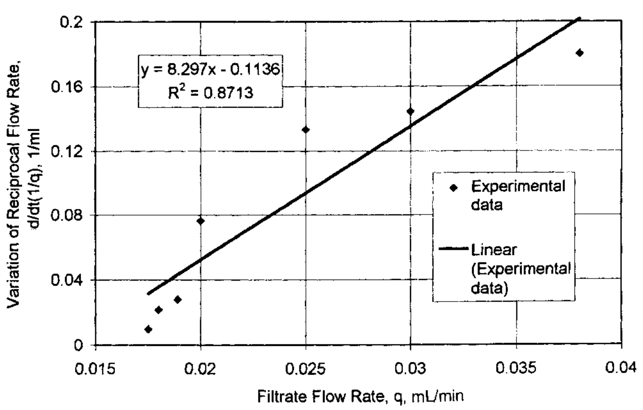

Figure 1. Correlation of Jiao and Sharma (1994) experimental data according to Eq. 52.

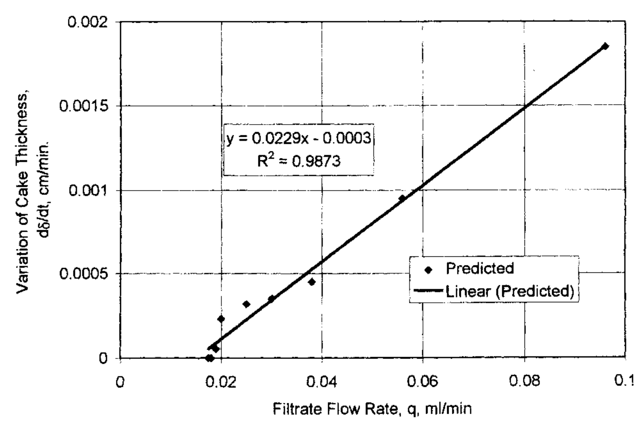

Figure 3. Correlation of Jiao and Sharma (1994) predicted filter cake thickness data according to Eq. 30.

$$A/C = 8.297 \text{ min/cm}^6,\ B/C = 0.1136 \text{ cm}^{-3},\ R^2 = 0.8713 \quad (60)$$

$$C = 0.0034 \text{ cm}^4\text{/min},\ D = 0.0076 \text{ cm},\ R^2 = 0.949 \quad (61)$$

$$A = 0.0229 \text{ cm}^{-2},\ B = 0.0003 \text{ cm/min},\ R^2 = 0.9873 \quad (62)$$

The coefficients of regressions very close to 1.0 indicate that the present equations closely represent the data. The coefficient of regression $R^2 = 0.8713$ indicated by Figure 1 and Eq. 60 is lower than those indicated by Figures 2 and 3, and Eqs. 61 and 62, inferring the possibility of larger measurement errors involved in the filtrate volume data. Another source of errors may be due to the three-point finite difference numerical differentiation of the filtrate volume data to obtain the filtrate flow rate data used to construct Figure 1. The data necessary for Figure 1 were obtained by a series of numerical procedures, first to calculate $q = dQ/dt$ from the filtrate volume Q data, and then $(1/q)$ and $[d/dt(1/q)]$.

The initial filtrate volume rate is obtained as $q_o = 0.096$ mL/min by a three-point forward differentiation of the measured, initial filtrate volume data. This data is expected to involve a larger error because of the possibility of relatively larger errors involved in the early filtrate volume data. The noisy data had to be smoothed prior to numerical differentiation, which may have introduced further errors. Because of the propagation of the significantly larger measurement errors involved in the early filtrate volume data, the first two of the $[d/dt(1/q)]$ values degenerated and deviated significantly from the expected straightline trend. Therefore, these two data points formed the outliers for linear regression and had to be discarded.

Substituting the values given in Eq. 60 into Eq. 55 yields the limiting filtrate flow rate as $q_\infty = 0.014$ mL/min. On the other hand, substituting the values given in Eq. 62 into Eq. 48 yields $q_\infty = 0.013$ mL/min. These two values obtained from the filtrate flow rate and cake thickness data, respectively, are very close to each other. The limiting filtrate volume rate q_∞ estimated by an extrapolation of the derivatives of the filtrate volume data beyond the range of the experimental data is $q_\infty = 0.017$ mL/min and close to the values obtained by the present regression method. This is an indication of the validity of the present filtration model.

Using $q_\infty = 0.014$ mL/min in Eq. 26 yields the limiting filter cake thickness as $\delta_\infty = 0.24$ cm. The predicted cake thickness data presented in Figure 10 of Jiao and Sharma (1994) indicates a value of approximately 0.17 cm. Therefore, their prediction of the limiting filter cake thickness appears to be an underestimation compared to the 0.24 cm value obtained by the present analysis.

The above obtained values can now be used to determine the values of the model parameters as follows. The filter cake permeability can be calculated by Eq. 50. Equations 46, 47, 51, 53 and 54 form a set of alternative equations to determine the deposition and erosion rate constants k_d and k_e. Here, Eqs. 46 and 51 were selected for this purpose. However, Jiao and Sharma (1994) do not offer any data on the cake porosity ϕ_c, and the critical shear stress τ_{cr} necessary for detachment of the particles from the progressing cake surface. Therefore, the ϕ_c and τ_{cr} parameters had to be estimated and used with Eqs. 46 and 51 to match the filtration

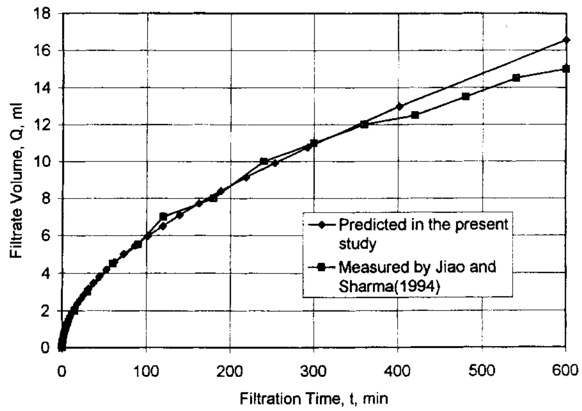

Figure 4. Predicted vs. measured filtrate volumes for linear filtration of fresh water bentonite suspension.

data over the period of the filtration process. Then, the ϕ_c and τ_{cr} values obtained this way were used in Eqs. 46 and 51 to calculate the k_d and k_e values.

Using the slurry tangential velocity of $\nu = 8.61$ cm/s, the typical particle diameter of $d = 2.5 \times 10^{-4}$ cm and the particle separation distance of $l = 2. \times 10^{-7}$ cm in Eq. 5, the critical shear stress for particle detachment is estimated to be $\tau_{cr} = 1.25 \times 10^3$ dyne/cm^2, whereas, the prevailing shear stress calculated by Eq. 33 is $\tau = 16$ dyne/cm^2. Under these conditions, theoretically, the cake erosion should not occur, because $\tau \ll \tau_{cr}$. Therefore, the value of the coefficient B should be zero. In contrast, as indicated by Eq. 62, the present analysis of the data has led to a small, but nonzero value of $B = 3. \times 10^{-4}$ cm/min. Recall that we used this value in Eq. 48 to calculate the limiting flow rate of $q_\infty = 0.013$ mL/min. This value was shown to be very close to the $q_\infty = 0.014$ mL/min value calculated by Eq. 55 and the approximate value of $q_\infty = 0.017$ mL/min obtained by extrapolating the filtrate flow rate data beyond the range of the experimental data. Thus, it is reasonable to assume that $B = 3. \times 10^{-4}$ cm/min is a meaningful value and not just a numerical result of the least-squares regression of Eq. 30 to data, because the coefficient of regression $R^2 = 0.9873$ is very close to one. Hence, it can be inferred that $\tau > \tau_{cr}$ and the cake erosion occurred in the actual experimental conditions of Jiao and Sharma (1994). In view of this discussion, it becomes apparent that the theoretical value obtained by Eq. 5 is not realistic.

The Jiao and Sharma (1994) data and the missing parameter values which have been approximated by fitting the experimental data are given in Table 1. The results presented in Figure 4 indicate that the model represents the measured filtrate volumes over the complete range of 600 min of filtration time as closely as the quality of their experimental data permits. However, they did not measure the cake thickness, but predicted it using a simple algebraic model. As shown in Figure 5, the cake thicknesses predicted by Jiao and Sharma (1994) and the present study are close to each other.

Willis et al. (1983) conducted linear filtration experiments using a suspension of lucite in water. As shown in Table 1, they reported only a few parameter values. They only provide some measured filtrate flow rate and cake thickness data in

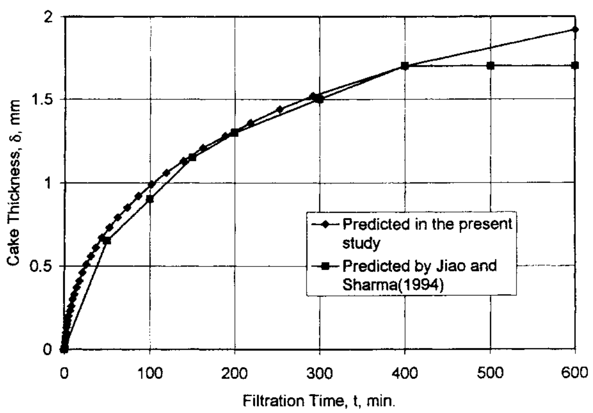

Figure 5. Predicted cake thicknesses for linear filtration of fresh water bentonite suspension.

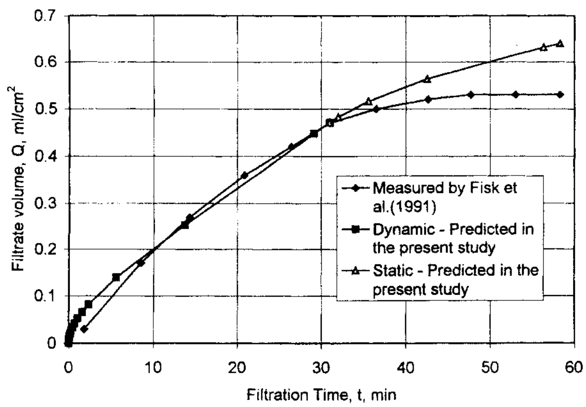

Figure 7. Predicted vs. measured filtrate volumes for radial filtration of a sea-water based partially hydrolyzed polyacrylamide drilling mud.

their Table 2. However, the filtration time data is missing. Therefore, a full-scale simulation of their filtration process as a function of time could not be carried out. Only the linear plotting of the measured data according to Eq. 26 could be accomplished. As indicated by Figure 6, the best linear fit of Eq. 26 with the least-squares method has been obtained with a coefficient of regression of $R^2 = 0.9921$ very close to 1.0. This reconfirms the validity of the present filtration model.

Fisk et al. (1991) conducted radial filtration experiments using a seawater-based partially hydrolyzed polyacrylamide mud. Their Figure 4 provides the measured dynamic and static filtrate volumes vs. filtration time data. Judging by their Figure 4, their static filtration data contains only three distinct measured values. This data is insufficient to extract a meaningful information on the values of the A and C coefficients by regression of Eq. 56, because the calculation of $\ln[-q^{-3}\ dq/dt]$ requires a two step, sequential numerical differentiation, first to obtain the filtrate flow rate $q = dQ/dt$ by differentiating the filtrate volume Q, and then differentiating q to obtain dq/dt. On the other hand, their dynamic filtration data is limited to the filtrate volume. As explained in the previous section on the determination of parameters, the determination of all coefficients of A, B, C, and D by means of Eqs. 10 and 20 requires both the filtration volume and filter cake thickness measurements. Therefore, the Fisk et al. (1991) radial filtration data has more missing parameter values which had to be approximated as given in Table 1. Figure 7 shows that the model predicts the measured dynamic and static filtrate volumes with reasonable accuracy in view of the uncertainties involved in the estimated values of the missing data. Fisk et al. (1991) did not report any results on the filter cake thickness and, therefore, a comparison of the cake thicknesses could not be made in the radial filtration case.

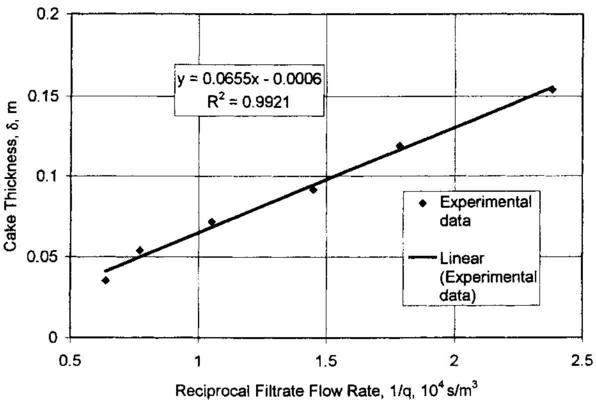

Figure 6. Correlation of Willis et al. (1983) measured filter cake thickness data according to Eq. 26.

Conclusion

The improved models developed in this study offer practical means of interpreting experimental data, estimating the model parameters, and simulating the linear and radial, incompressive cake filtration processes at static and dynamic filtration conditions. The simplified forms of the new models conform with the well-recognized simplified models reported in the literature. The new models are capable of capturing the responses of typical laboratory filtration tests while providing insight into the governing mechanisms.

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Notation

a = cross-sectional area of the filter, cm^2
A, B, C, D = parameters defined by Eqs. 11, 12, 18 and 19 for the radial model and by Eqs. 31, 32, 27 and 28 for the linear model, respectively, cm^{-2}, cm/s, cm^4/s, cm
h = filter width, cm
d = particle diameter, cm
L_f = filter length, cm
n' = flow index
r = radial distance, cm
μ = viscosity of fluids, cp
x = linear distance, cm

Subscripts

c = cake or slurry side cake surface
d = deposition
e = erosion or effluent side filter surface
p = particle
w = slurry side filter surface

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